The Effects of Superhigh Magnetic Fields on Equations of States of Neutron Stars

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By introducing Dirac δ-function in superhigh magnetic field, we deduce a general formula for pressure of degenerate and relativistic electrons, P_e, which is suitable for superhigh magnetic fields, discuss the quantization of Landau levels of electrons, and consider the quantum electrodynamic(QED) effects on the equations of states (EOSs) for different matter systems. The main conclusions are as follows: the stronger the magnetic field strength, the higher the electron pressure becomes; compared with a common radio pulsar, a magnetar could be a more compact oblate spheroid-like deformed neutron star due to the anisotropic total pressure; and an increase in the maximum mass of a magnetar is expected because of the positive contribution of the magnetic field energy to the EOS of the star. Since this is an original work in which some uncertainties could exist, to further modify and perfect our theory model should be considered in our future studies.

1 Introduction

Pulsars are among the most mysterious objects in the universe that provide natural laboratory for investigating the nature of matter under extreme conditions, and are universally recognized as normal neutron stars (NSs), but sometimes have been argued to be quark stars (Du et al. 2009, Lai et al. 2013, Xu et al. 2013). The equation of state (EoS) of matter under exotic conditions is an important tool for understanding of the nuclear force and for astrophysical applications. The Fermi energy of relativistic electrons $E_{\rm F}(e)$ is one of most important and indispensable physical parameters in EoS, and affects direct weak-interaction processes including modified URCA reactions, electron capture (e.g., Gao et al. 2011a, 2011b, 2011c, 2011d, 2012a, 2012b; Liu 2012, 2013, 2014, 2015; Du et al. 2014). These influences will change intrinsic EoS, interior structure and heat evolution, and even affect the whole properties of the star.

As we know, for degenerate and relativistic electrons in $\beta-$equilibrium, the distribution function $f(E_e)$ obeys Fermi-Dirac statistics: $f(E_e) = 1/(Exp((E_e - \mu_e)/kT) + 1)$, k represents Boltzmann's constant, and μ_e is the electron chemical potential. If $T \to 0$, μ_e is also called the electron Fermi energy, $E_{\rm F}(e)$, which presents the energy of highest occupied states for electrons. The electron Fermi energy $E_{\rm F}(e)$ has the simple form

$$E_{\rm F}(e) = \left(p_{\rm F}^2(e)c^2 + m_e^2c^4\right)^{1/2} \quad , \tag{1}$$

with $p_{\rm F}(e)$ being the electron Fermi momentum.

In the context of general relativity principle, the matter density is defined as: $\rho = \varepsilon/c^2$, ε is the total energy density, including the rest-mass energies of particles. Using the basic thermodynamics, we obtain the relation of the total matter pressure P and matter density ρ in a common NS,

$$\begin{aligned} &P(n_B) = n_B^2\frac{d(\varepsilon/n_B)}{dn_B} \ , \\ &\rho(n_B) = \varepsilon(n_B)/c^2, \quad \Rightarrow P = P(\rho) \ . \end{aligned} \tag{2}$$

From the above equation, it is obvious that P solely depends on ρ. Theoretically, we can obtain the value of $E_{\rm F}(e)$ by solving EOS in a specific matter model. The pressure of degenerate and relativistic electrons, P_e, is another important and indispensable physical parameter in EoSs of a NS. P_e is one of important dynamical pressures against a NS's gravitational collapse, and affects the structures and properties of the star, substantially.

Thompson and Duncan (1996) predicted that superhigh magnetic fields (MFs) could exist in the interiors of magnetars with a typical surface dipolar MF, $B \sim 10^{14}$ to 10^{15} G (Thompson & Duncan 1996). Superhigh MFs have effects on EoSs of a NS, as well as on its spin-down evolution (e.g., Gao et al. 2014, 2015). Recently, Franzon et al. (2015) studid the effects of strong MFs on hybrid stars by using a full

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general-relativity approach, and pointed that the MF could cause the stellar central density to be reduced, inducing major changes in the populated degrees of freedom and, potentially, converting a hybrid star into a hadronic star. In accordance with the popular point of view, the stronger the MF strength, the lower the electron pressure becomes. With respect to this view, we cannot directly verify it by experiment in actual existence, owing to lack of such high MFs on the earth. After a careful check, we found that popular methods of calculating $E_F(e)$ the electron Fermi energy are contradictory to the quantization of electron Landau levels. In an extremely strong MF, the Landau column becomes a very long narrow cylinder along MF, If we consider Dirac δ-function in superhigh MFs, all the results should be reconsidered.

In Sec. 2, we deduce an equation of P_e in superhigh MFs; in Sec. 3, we consider QED effects on EOSs of NS matter, and discuss an anisotropy of the total pressure; In Sec. 4 we discuss our future work on improving our model, and present conclusions in Sec. 5.

2 Deduction of the pressure of electrons in superhigh MFs

The relativistic Dirac-Equation for the electrons in a uniform external magnetic field along the $z-$axis gives the electron energy level

$$E_e = [m_e^2c^4(1+\nu\frac{2B}{B_{\rm cr}})+p_z^2c^2]^{\frac{1}{2}} \ , \quad (3)$$

where $\nu = n+\frac{1}{2}+\sigma$ is the quantum number, n the Landau level number, $\sigma = \pm\frac{1}{2}$ the spin quantum number (Canuto & Ventura 1977), and p_z is the z-component of electron momentum, and may be treated as a continuous function. Combining $B_{\rm cr} = m_e^2c^3/e\hbar$ with $\mu_e^{'} = e\hbar/2m_ec$ gives

$$\begin{aligned} E_e^2 \ &= \ m_e^2c^4 + p_z^2c^2 + 2\nu 2m_ec^2\mu_e^{'}B \\ &= m_e^2c^4 + p_z^2c^2 + p_\perp^2c^2 \ , \end{aligned} \quad (4)$$

where $\mu_e^{'}$ is the magnetic moment of an electron, and $p_\perp = m_ec(2\nu B^*)^{\frac{1}{2}}$). The maximum electron Landau level number n_{max} is uniquely determined by the condition $[p_{\rm F}(z)c]^2 \geq 0$ (Lai & Shapiro 1991, Gao et al. 2013), where $p_{\rm F}(z)$ is the Fermi momentum along the $z-$axis. The expression for ν_{max} can be expressed as

$$\begin{aligned} &\nu_{max}(\sigma = -\frac{1}{2}) \\ &= Int[\frac{1}{2B^*}[(\frac{E_{\rm F}(e)}{m_ec^2})^2 - 1 - (\frac{p_z}{m_ec})^2] + \frac{1}{2} - \frac{1}{2}] \\ &= Int[\frac{1}{2B^*}[(\frac{E_{\rm F}(e)}{m_ec^2})^2 - 1 - (\frac{p_z}{m_ec})^2]] \ , \\ &\nu_{max}(\sigma = \frac{1}{2}) \\ &= Int[\frac{1}{2B^*}[(\frac{E_{\rm F}(e)}{m_ec^2})^2 - 1 - (\frac{p_z}{m_ec})^2] - 1 + \frac{1}{2} + \ \frac{1}{2}] \\ &= Int[\frac{1}{2B^*}[(\frac{E_{\rm F}(e)}{m_ec^2})^2 - 1 - (\frac{p_z}{m_ec})^2]] \ . \end{aligned} \quad (5)$$

According to the definition of $E_{\rm F}(e)$ in Eq.(1), we obtain $E_{\rm F}(e) \ \equiv \ p_{\rm F}(e)c$ if electrons are super-relativistic ($E_{\rm F}(e) \ \gg \ m_ec^2$). In the presence of a superhigh MF, $B \gg B_{\rm cr}$, $E_{\rm F}(e) \gg m_ec^2$), we have

$$\begin{aligned} &\nu_{max}^{'}(\sigma = -\frac{1}{2}) = \nu_{max}^{'}(\sigma = \frac{1}{2}) \\ &\simeq Int[\frac{1}{2B^*}[(\frac{E_{\rm F}(e)}{m_ec^2})^2] \ . \end{aligned} \quad (6)$$

The maximum of $p_\perp$ for electrons in a superhigh MF is

$$p_\perp^2(max)c^2 = \ 2\nu_{max}^{'}2m_ec^2\mu_e^{'}B \ \ , \quad (7)$$

where the relation of $2\mu_e^{'}B_{\rm cr}/m_ec^2 = 1$ is used. Inserting Eq.(6) into Eq.(7) gives

$$\begin{aligned} &p_\perp^2(max)c^2 = \ 2\times\frac{1}{2B^*}(\frac{E_{\rm F}(e)}{m_ec^2})^2\times 2m_ec^2\mu_e^{'}B \\ &\simeq B_{\rm cr}\times(\frac{E_{\rm F}(e)}{m_ec^2})^2\times 2m_ec^2\frac{e\hbar}{2m_ec} \\ &= \frac{m_e^2c^3}{e\hbar}\times(\frac{E_{\rm F}(e)}{m_ec^2})^2\times 2m_ec^2\frac{e\hbar}{2m_ec} = \ E_{\rm F}^2(e) \ . \end{aligned} \quad (8)$$

In superhigh MFs, $E_{\rm F}(e)$ is determined by

$$E_{\rm F}(e) \simeq 43.44(\frac{B}{B_{\rm cr}})^{1/4}(\frac{\rho}{\rho_0}\frac{Y_e}{0.0535})^{\frac{1}{4}} \ {\rm MeV} \ , \quad (9)$$

where $\rho_0 = 2.8\times10^{14}$ g cm^3 is the standard nuclear density (Gao et al. 2012b). Thus, we obtain

$$\begin{aligned} &p_\perp(max) = p_{\rm F}(e) \simeq \frac{E_{\rm F}(e)}{c} \\ &= 43.44\times(\frac{Y_e}{0.0535}\frac{\rho}{\rho_0}\frac{B}{B_{\rm cr}})^{\frac{1}{4}}{\rm MeV/c} \ (B^* \geq 1\ ) \end{aligned} \quad (10)$$

As pointed out above, when $n = 0$, the electron Landau level is non-degenerate, and p_z has its maximum $p_z(max)$,

$$\begin{aligned} &p_z(max) = p_{\rm F}(e) \simeq \frac{E_{\rm F}(e)}{c} \\ &= 43.44\times(\frac{Y_e}{0.0535}\frac{\rho}{\rho_0}\frac{B}{B_{\rm cr}})^{\frac{1}{4}}{\rm MeV/c} \ (B^* \geq 1\ ) \end{aligned} \quad (11)$$

From Eq.(9) and Eq.(10), it's obvious that $p_z(max) = p_\perp(max) = p_{\rm F}(e)$. The reason for this is that in the interior of a magnetar, electrons are degenerate and super-relativistic, and can be approximately treated as an ideal Fermi gas with equivalent pressures in all directions, though the existence of Landau levels. The equation of P_e in a superhigh magnetic field is consequently given by

$$\begin{aligned} P_e &= \frac{1}{3}\frac{2}{h^3}\int_0^{p_{\rm F}(e)}\frac{p_e^2c^2}{(p_e^2c^2+m_e^2c^4)^{1/2}}4\pi p_e^2dp_e \\ &= 1.412\times10^{25}\phi(x_e){\rm dynes\ cm}^{-2} \end{aligned} \quad (12)$$

where $\lambda_e = \frac{h}{m_ec}$ is the electron Compton wavelength, $x_e = \frac{p_{\rm F}(e)}{m_ec} \simeq \frac{E_{\rm F}(e)}{m_ec^2} = 86.77\times(\frac{\rho}{\rho_0}\frac{B}{B_{\rm cr}}\frac{Y_e}{0.0535})^{\frac{1}{4}}$), and $\phi(x_e)$ is the polynomial $\phi(x_e) = \frac{1}{8\pi^2}[x_e(1+x_e^2)^{\frac{1}{2}}(\frac{2x_e^2}{3}-1)+ln[x_e+(1+x_e^2)^{\frac{1}{2}}]]$.

When $\rho \geq 10^7$ g cm^{-3}, $x_e \gg 1$, and $\phi(x_e) \to \frac{x_e^4}{12\pi^2}$. Thus, Eq.(11) can be rewritten as

$$P_e \simeq 6.266\times10^{30}(\frac{\rho}{\rho_0}\frac{B}{B_{\rm cr}}\frac{Y_e}{0.0535}) \ {\rm dyne\ cm^{-2}}. \quad (13)$$

It is obvious that P_e increases sharply with increasing B when the values of Y_e and ρ are given.

3 Magnetic effects on EoSs

3.1 Magnetic effects on the EoS of BPS model

By introducing the lattice energy, Baym, Pethick & Sutherland (1971) (hereafter "BPS model") improved on Salpeter's treatment (Salpeter 1961), and described the nuclear composition and EoS for catalyzed matter in complete thermodynamic equilibrium below ρ_d. BPS model is one of most successful models describing matter of the outer crust. According to BPS model, the matter energy density is given by

$$\varepsilon = n_N(W_N(A,Z) + \varepsilon_L(Z,n_e) + \varepsilon_e(n_e)\,, \tag{14}$$

where n_N is the number density of nuclei, $W_N(A,Z)$ is the mass- energy per nucleus (including the rest mass of Z electrons and A nucleons); ε_e is the free electron energy including the rest mass of electrons in a unit volume; ε_L is the *bcc* Coulomb lattice energy per nucleus,

$$\varepsilon_L = -1.444 Z^{2/3} e^2 e^2 n_e^{4/3}\ , \tag{15}$$

where the relations of $n_N = n_B/A$ and $n_e = Zn_N$ are used. The matter pressure p of the system is given by

$$P = P_e + P_L = P_e + \frac{1}{3}\varepsilon_L. \tag{16}$$

For a magnetic field $B^* \gg 1$, P_e in Eq.(15) is given by Eq.(12). Based on the above equations, we plot one schematic diagrams of QED effects on the EOS of BPS model, as shown in Fig.1.

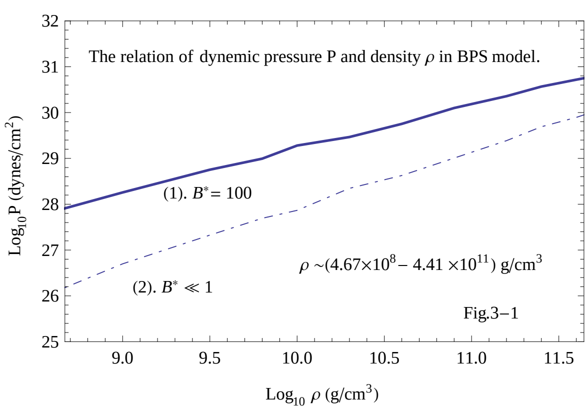

Fig. 1 P vs. ρ of BPS model below neutron drip.

3.2 The QED effects on the EOS of ideal npe gas

We consider a homogenous ideal npe gas under β- equilibrium, and adopt ST83 approximation (Shapiro & Teukolskysee 1983) corresponding to the weak-field limit as the main method to treat EoS of this system in the density range of $0.5 \sim 2.5\rho_0$ where electrons are relativistic, neutrons and protons are non-relativistic. When neutron pressure dominates, $\rho \approx m_n n_n$, then $n_n = 1.7 \times 10^{38}(\frac{\rho}{\rho_0})$ cm^{-3} (Shapiro & Teukolskysee 1983); employing β- equilibrium and charge neutrality gives $n_p = n_e = 9.6 \times 10^{35}(\frac{\rho}{\rho_0})^2$ cm^{-3}; β-equilibrium implies energy conservation and momentum conservation ($p_{\rm F}(p) = p_{\rm F}(e)$), we get $E_{\rm F}(e) = \mu_n = E'_{\rm F}(n) = p^2_{\rm F}(n)/2m_n = 60(\frac{\rho}{\rho_0})^{2/3}$ MeV, and $\mu_p = E'_{\rm F}(p) = p^2_{\rm F}(p)/2m_p = 1.9(\frac{\rho}{\rho_0})^{4/3}$ MeV; the isotropic matter pressure P is given by

$$P = P_e + P_p + P_n$$
$$= \frac{m_e c^2}{\lambda_e^3}\phi(x_e) + \frac{m_p c^2}{\lambda_p^3}\phi(x_p) + \frac{m_n c^2}{\lambda_n^3}\phi(x_n)\ , \tag{17}$$

where $x_p = \frac{p_{\rm F}(p)}{m_p c} = \frac{\sqrt{2m_p\mu_p}}{m_p c} = \sqrt{\frac{2\mu_p}{m_p c^2}}$, the expression of $\phi(x_p)$ is completely similar to that of $\phi(x_{\rm n})$.

Based on the above results, we gain the following useful formulae:

$$P_p = 1.169 \times 10^{30}(\frac{\rho}{\rho_0})^{\frac{10}{3}}\ \text{dynes cm}^{-2}\ ,$$
$$P_e = 1.825 \times 10^{31}(\frac{\rho}{\rho_0})^{\frac{8}{3}}\ \text{dynes cm}^{-2}\ ,$$
$$P_n = 6.807 \times 10^{33}(\frac{\rho}{\rho_0})^{\frac{5}{3}}\ \text{dynes cm}^{-2}\ ,$$
$$Y_e = \frac{n_e}{n_p + n_n} \simeq \frac{n_e}{n_n} = 0.005647(\frac{\rho}{\rho_0})\ . \tag{18}$$

Our methods to treat EOS of an ideal npe gas (system) under β-equilibrium in superhigh MFs are introduced as follows: Combining Eq.(9) with momentum conservation gives the chemical potential $\mu_p = E'_{\rm F}(p) = 1.005(\frac{B}{B_{\rm cr}}\frac{\rho}{\rho_0}\frac{Y_e}{0.0535})^{\frac{1}{2}}$ MeV, and the non-dimensional variable $x_p = \sqrt{\frac{2\mu_p}{m_p c^2}} \simeq 4.626 \times 10^{-2}(\frac{B}{B_{\rm cr}}\frac{\rho}{\rho_0}\frac{Y_e}{0.0535})^{\frac{1}{4}}$; Then we get

$$x_n = \sqrt{\frac{1}{m_n c^2}}(2 \times (43.44(\frac{B}{B_{\rm cr}}\frac{\rho}{\rho_0}\frac{Y_e}{0.0535})^{1/4}$$
$$-1.29 + 1.005(\frac{B}{B_{\rm cr}}\frac{\rho}{\rho_0}\frac{Y_e}{0.0535})^{1/2}))^{1/2}. \tag{19}$$

The β-equilibrium condition gives the expression for the isotropic matter pressure P,

$$P = \frac{m_e c^2}{\lambda_e^3}\phi(x_e) + \frac{m_p c^2}{\lambda_p^3}\phi(x_p) + \frac{m_n c^2}{\lambda_n^3}\phi(x_n)$$
$$= 6.266 \times 10^{30}(\frac{\rho}{\rho_0}\frac{B}{B_{\rm cr}}\frac{Y_e}{0.0535}) + 2.324 \times 10^{26}$$
$$\times(\frac{\rho}{\rho_0}\frac{B}{B_{\rm cr}}\frac{Y_e}{0.0535})^{\frac{5}{4}} + 1.624 \times 10^{38}$$
$$\times\frac{1}{15\pi^2}(x_n^5 - \frac{5}{14}x_n^7 + \frac{5}{24}x_n^9)\ \text{dyne cm}^{-2}\ , \tag{20}$$

where x_n is determined by Eq.(19). The above equation always approximately hold in an ideal npe gas when $B^* \gg 1$ and $\rho \sim 0.5\rho_0 - 2\rho_0$. Based on the above equations, we plot two schematic diagrams of QED effects on EOS of this npe gas, as shown in Figs.2-3. Both P_p and P_n increase obviously with ρ and B for an ideal npe gas.

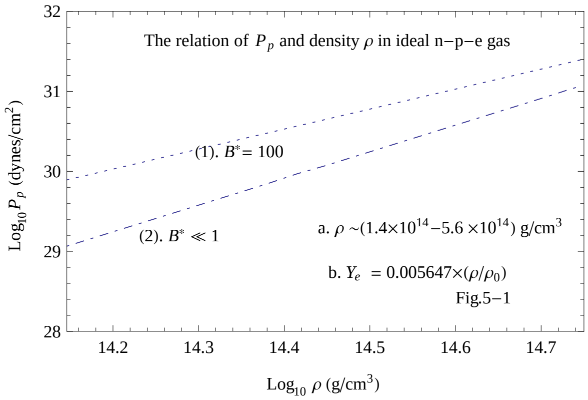

Fig. 2 P_p vs. ρ for an ideal npe gas in a superhigh MF.

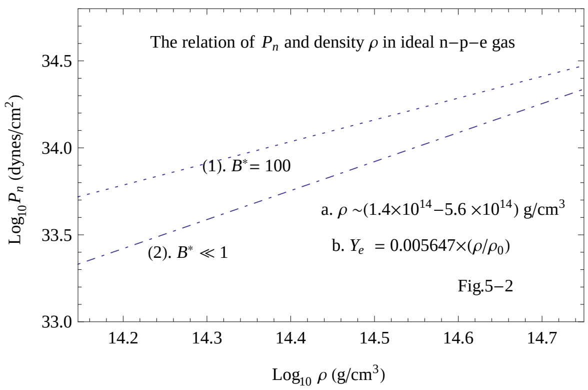

Fig. 3 P_n vs. ρ for an ideal npe gas in a superhigh MF.

3.3 The QED effects on the total matter pressure and total energy density

As discussed above, the pressures of fermions increase with B, the total matter pressure increases with B. Due to a positive co-relation between the total energy density ε and the total matter pressure, ε also increases with B.

The stable configurations of a NS can be obtained from the well-known hydrostatic equilibrium equations of Tolman, Oppenheimer and Volkov (TOV) for the pressure $P(r)$ and the enclosed mass $m(r)$,

$$\frac{dP(r)}{dr} = -\frac{G(m(r) + 4\pi r^3 P(r)/c^2)(\rho + P(r)/c^2)}{r(r - 2Gm(r)/c^2)}$$
$$\frac{dm(r)}{dr} = 4\pi\rho r^2 \ , \tag{21}$$

where G is the gravitational constant. For a chosen central value of ρ, the numerical integration of Eq.(21) provides the mass-radius relation. In Eq.(21), the pressure $P(r)$ is the gravitational collapse pressure, and always be balanced by the total dynamics pressure, P; the central density ρ is proportional to the matter energy density ϵ; the enclosed mass, $m(r)$, increases with the central density ρ when r is given.

As we know, the magnetic effects can give rise to an anisotropy of the total pressure of the system to become anisotropic (Bocquet et al. 1995, Paulucci et al. 2011). The total energy momentum tensor due to both matter and magnetic field is to be given by

$$T^{\mu\nu} = T^{\mu\nu}_m + T^{\mu\nu}_B, \tag{22}$$

where,

$$T^{\mu\nu}_m = \epsilon_m u^\mu u^\nu - P_m(g^{\mu\nu} - u^\mu u^\nu), \tag{23}$$

and

$$T^{\mu\nu}_B = \frac{B^2}{4\pi}(u^\mu u^\nu - \frac{1}{2}g^{\mu\nu}) - \frac{B^\mu B^\nu}{4\pi} \ . \tag{24}$$

The first term in Eq.(24) is equivalent to magnetic pressure, while the second term causes the magnetic tension. Due to an excess negative pressure or tension along the direction to the magnetic field, the component of $T^{\mu\nu}_B$ along the field, T^{zz}_B, is negative. Thus, the total pressure in the parallel direction to MF can be written as

$$P_\parallel = P_m - \frac{B^2}{8\pi}, \tag{25}$$

and that perpendicular to MF, $P_\perp$, is written as

$$P_\perp = P_m + \frac{B^2}{8\pi} - \mathcal{M}B \ , \tag{26}$$

where $\mathcal{M}$ is the magnetization of the system, and $\mathcal{M}B$ is the magnetization pressure (Perez et al. 2008, Ferrer et. al. 2010). In this work magnetars universally have typical dipole MFs $\sim (10^{14} - 10^{15})$ G and inner field strengths not more than 10^{17} G, under which the system magnetic moment satisfies $\mathcal{M} < B$, a condition that can be justified for any medium that is not ferromagnetic, the effect of AMMs of nucleons on the EOS are insignificant and thus ignored (Ferrer et. al. 2015). It's obvious that the total pressure of the system becomes anisotropic, that is $P_\perp > P_\parallel$, which could lead to the Earth-like oblatening effect.

According to our calculations, when $B^* = 100$, $P_m \sim 10^{33} - 10^{34}$ dynes cm^{-2} and $\frac{B^2}{8\pi} \sim 10^{29} - 10^{30}$. Hence, in this presentation, we consider that the component of the total energy momentum tensor along the symmetry axis becomes positive, $T^{zz} > 0$, since the total matter pressure increases more rapidly than the magnetic pressure. We propose that the component of the total energy momentum tensor along the symmetry axis becomes positive, since P_m always grows more rapidly than the magnetic pressure. The magnetic tension along the direction to the magnetic field will be responsible for deforming a magnetar along MF, and turns the star into a kind of oblate spheroid. Be note that such a deformation in shape might even render a more compact magnetar endowed with canonical strong surface fields $B \sim 10^{14-15}$ G. Also, such a deformed magnetar could have a more massive mass because of the positive contribution of the magnetic field energy to EOSs of a magnetar.

4 To modify P_e in superhigh MFs

According to atomic physics physics, the higher the orbit quantum number l is, the larger the probability of an electron's transition (this transition is referred to the transition

from a higher energy level into a lower energy level) is. Analogous to atomic energy level, in a superhigh MF, the easier an electron's transition from a higher Landau level into a lower Landau level, Thus, the higher the electron Landau level number n, the lower the stability of the Landau level. Owing to the uncertainties of microscopic states, we introduce a new quantity, g_n, the stability coefficient of electron Landau level in a superhigh MF, and assume that g_n decreases with n as an exponential form,

$$g_n = g_0 n^{\alpha}, \tag{27}$$

where g_0 is the stability coefficient of the ground-state Landau level of electrons, α is the Landau level stability index, and is restricted to be $\alpha < 0$. From Eq.(27), it is obvious that $g(n)$ is a function of n and α, and the higher n is, the smaller g_n is (except for $g_1 = g_0$).

According to the Pauli exclusion principle, electron energy state number in a unit volume, N_{pha}, should be equal to electron number in a unit volume, n_e. Considering the electron Landau level stability coefficient g_n, and summing over electron energy states in a 6-dimension phase space, we can express N_{pha} as follows:

$$\begin{aligned} N_{pha} &= n_e = N_A \rho Y_e \\ &= \frac{2\pi}{h^3}\int dp_z \sum_{n=0}^{n_m(p_z,\sigma,B^*)} \sum g_n \\ &\times \int \delta(\frac{p_\perp}{m_e c} - [(2n+1+\sigma)B^*]^{\frac{1}{2}}) p_\perp dp_\perp, \end{aligned} \tag{28}$$

where N_A is the Avogadro constant. When $n_m \geq 6$, the summation formula can be approximately replaced by the following integral equation

$$\sum_{n=0}^{n_m} n^{\alpha+\frac{1}{2}} \simeq \int_0^{n_m} n^{\alpha+\frac{1}{2}} dn = \frac{2}{2\alpha+3} n_m^{\alpha+\frac{3}{2}}. \tag{29}$$

Thus, Eq.(28) can be rewritten as

$$\begin{aligned} N_{pha} &= N_A \rho Y_e = \frac{2^{\frac{7}{2}}}{2\alpha+3}\pi\sqrt{B^*}(\frac{m_e c}{h})^3 g_0 \\ &\int_0^{\frac{p_F}{m_e c}} [(\frac{E_F(e)}{m_e c^2})^2 - 1 - (\frac{p_z}{m_e c})^2]^{\alpha+\frac{3}{2}} d(\frac{p_z}{m_e c}). \end{aligned} \tag{30}$$

After a complicate deduction process, we get an non-dimensional momentum

$$x_e = \frac{p_F(e)}{m_e c} = C\left[\frac{Y_e}{0.05}\frac{\rho}{\rho_0}\right]^{\frac{1}{2(\alpha+2)}} (B^*)^{\frac{\alpha+1}{2(\alpha+2)}}, \tag{31}$$

where C is a constant, which is determined by

$$\begin{aligned} C &= (\frac{0.05\rho_0 N_A(2\alpha+3)}{2^{2(1-\alpha)}\pi g_0 I(\alpha)})^{\frac{1}{2(\alpha+2)}} (\frac{h}{m_e c})^{\frac{3}{2(\alpha+2)}} \\ &\simeq (337.12)^{\frac{3}{2(\alpha+2)}} (\frac{2\alpha+3}{2^{2(1-\alpha)} g_0 I(\alpha)})^{\frac{1}{2(\alpha+2)}}. \end{aligned} \tag{32}$$

with $\int_0^1 (1-t^2)^{3/2+\alpha}$, and $t = p_z c/E_F(e)$. If α and g_0 are determined, the expressions of $E_F(e)$ and P_e in superhigh MFs will be modified accordingly. To exactly determine the values of α and g_0 is an interesting and important task, but is beyond of this paper. Since this is an original work in which some uncertainties could exist, to further modify and perfect our model should be considered in our future studies, especially to further investigate QED effects on the EoSs using an improved expression of P_e in a superhigh MF.

5 Conclusions

In this presentation, we derived a general expression for electron pressure, which holds in a superhigh MF, considered QED effects on EoSs of neutron star matter, and discussed an anisotropy of the total pressure in superhigh MFs. Compared with a common pulsar, a magnetar could be a more compact oblate spheroid-like deformed NS, due to the anisotropic total pressure; an increase in the maximum mass of a magnetar is expected because of the positive contribution of the magnetic field energy to EoS.

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